Practice Problem Series

ADDITIONAL QUESTIONS ON CHAPTER 01 – 05

Q01. (a) Find the radian measure of $-106^{\circ}29'60''$. (b) Find the degree measure of 16/5.

Q02. In a right angled triangle, the difference between two acute angles is $\pi/15$. Find the remaining angles in degree.

Q03. If $\operatorname{cosec} x = -\frac{5}{2\sqrt{6}}$, $x \in$ III quadrant, find the remaining trigonometric functions.

Q04. Evaluate the followings:

(a) $\sin 75^{\circ}$ (b) $\tan\left(-\frac{16\pi}{3}\right)$

(c) $\cos\frac{7\pi}{12}$ (d) $\sin(-765^{\circ})$.

Q05. Prove the followings:

(a) $\frac{\tan(45^{\circ}+x)}{\tan(45^{\circ}-x)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$

(b) $\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x+\tan y}{\tan x-\tan y}$

(c) $\frac{\cos x}{1-\sin x} = \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$.

Q06. If $\tan x = -4/3$ such that $\frac{\pi}{2} < x < \pi$, find the values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\cot\frac{x}{2}$.

Q07. Solve the followings:

(a) $2\sin^2 x + \sqrt{3}\cos x + 1 = 0$ (b) $2\cos^2 x + 3\sin x = 0$.

Q08. By using induction, prove that: $1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = n^2(2n^2-1)$.

Q09. Let A = {1, 2, 3, …, 14}. Define a relation R from set A to A such that R = {(x, y): $3x - y = 0$; $x, y \in$ A}. Write its domain, range and co-domain.

Q10. Examine if the relation g = {(2,1), (4,2), (6,3), (8,4), (10,5), (12,6)} is a function or not. Justify.

Q11. Find the domain of $\frac{x}{\sqrt{x^2-3x+2}}$.

Q12. Find the range of $f(x) = \frac{x-8}{8-x}$.

Q13. Write the subset and power set for $\{\phi, \{1,2\}\}$.

Q14. Show that $A \cup B = A \cap B$ implies $A = B$.

Q15. Simplify: $\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+\sqrt{-16}}{2-\sqrt{-16}}\right)$.

Q16. Find the multiplicative inverse and additive inverse of $-\sqrt{3}+\sqrt{-2}$.

Q17. Find the modulus, argument and the conjugate of $\frac{2-i}{4i+(i+1)^2}$.

Q18. Find the polar form of the following:

(a) $\frac{16}{1+\sqrt{-3}}$ (b) $\frac{1-i}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$.

Q19. If $\sqrt[3]{x+iy} = a+ib$ then, show that $\frac{x}{a}+\frac{y}{b} = 4(a^2-b^2)$.

Q20. For any two complex numbers z_1 and z_2 prove that $\text{Im}(z_1z_2) = \text{Re}z_1\text{Im}z_2 + \text{Im}z_1\text{Re}z_2$.

Q21. Find $\sqrt{12-5i}$.

Q22. If $\sin x = 3/5$, $\cos y = -12/13$; $x, y \in$ II quadrant, find the value of $\cos(x+y)$.

Q23. Show that $2.7^n + 3.5^n - 5$ is divisible by 24.

Q24. If A and B are two sets containing 3 and 6 elements respectively, what can be the maximum number of elements in $A \cup B$?

Q25. Prove that: $\cos 5x = 16\cos^5 x - 20\cos^3 x + 5\cos x$.

❖ ANSWERS ❖

Q01. (a) $-\frac{71\pi}{120}$ (b) $183^\circ 16' 21''$ (Approx.) Q02. $51^\circ, 39^\circ$

Q03. $\sin x = -\frac{2\sqrt{6}}{5}, \cos x = -\frac{1}{5}, \sec x = -5, \tan x = 2\sqrt{6}, \cot x = \frac{1}{2\sqrt{6}}$

Q04. (a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (b) $-\sqrt{3}$ (c) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ (d) $-\frac{1}{\sqrt{2}}$

Q06. $\sin\frac{x}{2} = \frac{2}{\sqrt{5}}, \cos\frac{x}{2} = \frac{1}{\sqrt{5}}$ and $\cot\frac{x}{2} = \frac{1}{2}$

Q07. (a) $2n\pi \pm \frac{5\pi}{6}, n \in Z$ (b) $n\pi + (-1)^n \frac{7\pi}{6}, n \in Z$

Q09. R = {(1,3), (2,6), (3,9), (4,12)}. Domain = {1, 2, 3, 4}, Range = {3, 6, 9, 12} and co-domain = A.

Q10. Since the first components of ordered pairs belonging to *g* are 2, 4, 6, 8, 10, 12, 14 which are different and have distinct images *i.e.*, different second components of ordered pairs hence, *g* is a function. Q11. $(-\infty, 1) \cup (2, \infty)$. Q12. $\{-1\}$.

Q13. Subsets: $\phi, \{\phi\}, \{\{1,2\}\}, \{\phi, \{1,2\}\}$ and power set: $\{\phi, \{\phi\}, \{\{1,2\}\}, \{\phi, \{1,2\}\}\}$.

Q15. $\frac{1}{4} + i\frac{9}{4}$. Q16. $-\frac{\sqrt{3}}{5} - \frac{\sqrt{2}}{5}i$; $\sqrt{3} - \sqrt{2i}$

Q17. $\frac{\sqrt{5}}{6}, \tan^{-1}2 - \pi, -\frac{1}{6} + \frac{1}{3}i$ Q18. (a) $8\left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right]$

(b) $\sqrt{2}\left[\cos\left(-\frac{5\pi}{12}\right) + i\sin\left(-\frac{5\pi}{12}\right)\right]$

Q21. $\pm\left(\frac{5}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$ Q22. 33/65 Q24. 9.